Design and Optimization of an Optical Cavity to Interact with ^{87}Rb

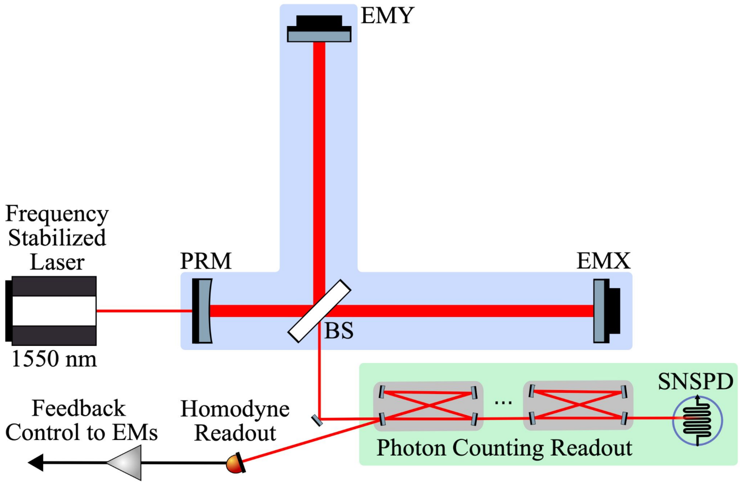
Rafael Crespo
Mentors: Daniel Grass and Professor Lee McCuller
California Institute of Technology

Introduction/Background

- I have been working on optimizing an optical cavity based on a new "photon counting" technique that will provide the necessary sensitivity to better detect weak signals.

- This apparatus, which will be able to distinguish between the desired signal and other sources of noise, utilizes an atomic cavity.

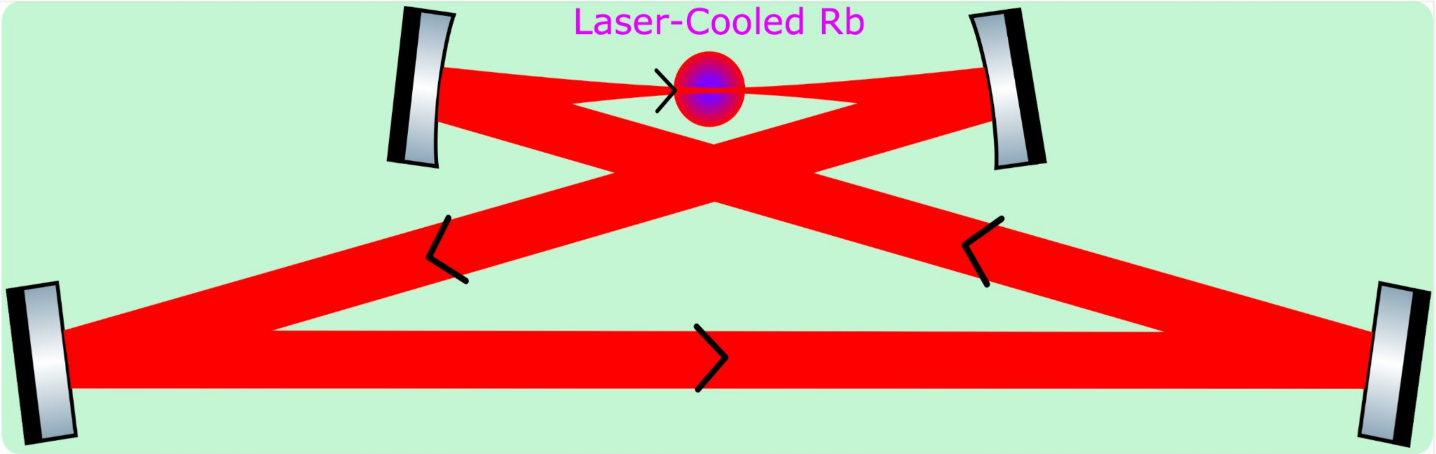
- Using atoms in a laser cavity, rather than just the laser cavity by itself, will reduce signal loss and improve the filtering of the incoming light.



Vermeulen et al., 2024

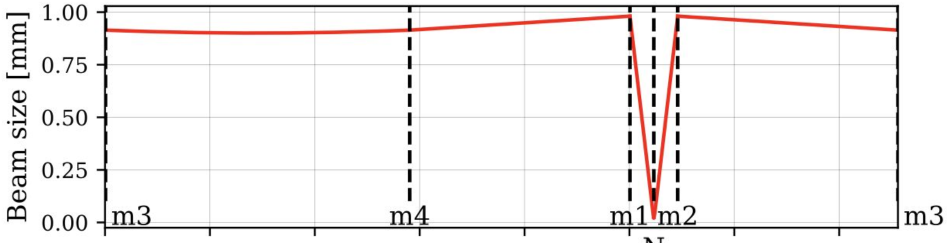
Introduction/Background

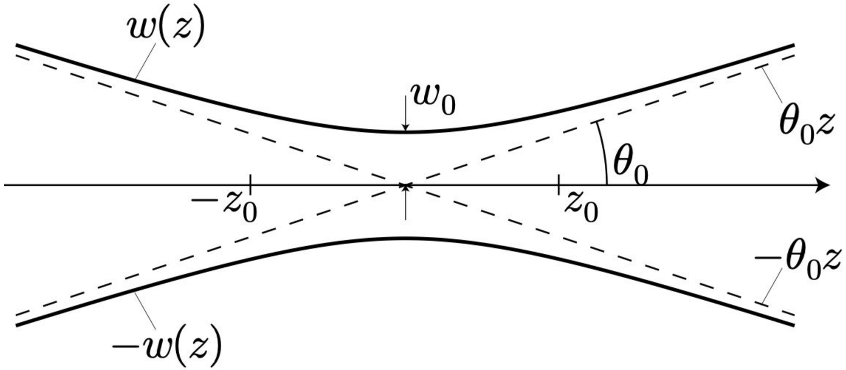
- The readout method involves measuring the state of the valence electron of the Rb atom after its interaction with the IFO output signal.
- We want to increase the probability of an interaction between the incoming photon and an Rb atom.
- To do this, we will use a cavity that goes through a laser-cooled cloud of Rubidium atoms in a 4-mirror setup.
- Two of the mirrors will be curved in order to focus the incoming laser beam to as small a waist as possible.



Methods

- In practice, I used Finesse 3 to simulate the laser cavity.
- Given optical elements and path (assuming the beam is stable), it propagates the beam along its defined path, calculating important beam characteristics (w, w0, z, zR, and others).
- Parametrized by distance between mirrors and angle of incidence (AOI) (but with some assumptions to simplify geometry), as well as the mirror's radius of curvature (RoC).

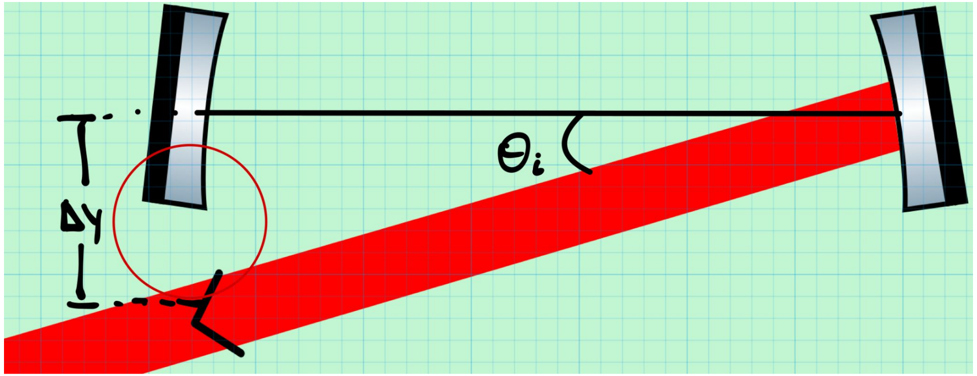




Steck, 2024

Methods

- The waist depends on the angle of incidence.
- Ideally we want the AOI = 0º.
- The minimum angle of incidence is limited in practice by clipping.
- This means that the edge of the mirrors should far away from the propagating beam's center.
- Thus I approximated the segment from the center of the beam to the bottom of M1 and that from the bottom of M1 to its center to be parallel.



$$I(r,z) = I_0 \left[\frac{w_0}{w(z)}\right]^2 \exp\left[-\frac{2r^2}{w^2(z)}\right]$$

Steck, 2024

Methods

- Given that constraint, the AOI was calculated using simple trigonometry:

$$\tan\theta_i = \frac{\Delta y}{L_1}$$
$$\theta_i = \arctan\frac{\Delta y}{L_1}$$
$$= \arctan\frac{20.2\ \mathrm{mm}}{180\ \mathrm{mm}}$$
$$= 3.20^\circ.$$

- This produced the following beam in Finesse 3:

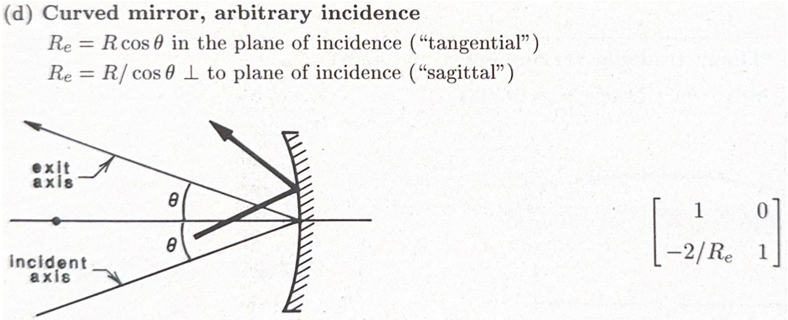
	z	w0	zr	w
L0.p1.o	0 m	900.52 um	1.6436 m	914.52 um
m3.p1.i	1 mm	900.52 um	1.6436 m	914.43 um
m3.p3.o	1 mm	900.52 um	1.6436 m	914.43 um
m4.p1.i	581 mm	900.52 um	1.6436 m	914.43 um
m4.p2.o	581 mm	900.52 um	1.6436 m	914.43 um
m1.p4.i	1.001 m	900.52 um	1.6436 m	980.95 um
m1.p3.o	1.001 m	22.891 um	1.0621 mm	980.95 um
N.p1.i	1.0465 m	22.891 um	1.0621 mm	22.891 um
N.p2.o	1.0465 m	22.891 um	1.0621 mm	22.891 um
m2.p1.i	1.092 m	22.891 um	1.0621 mm	980.95 um
m2.p2.o	1.092 m	900.52 um	1.6436 m	980.95 um
m3.p4.i	1.512 m	900.52 um	1.6436 m	914.43 um

Methods

- Finally, I saw that there is astigmatism, which makes the waist size different depending on the axis.

- Thus, actual effective waist size is between that of each axis.

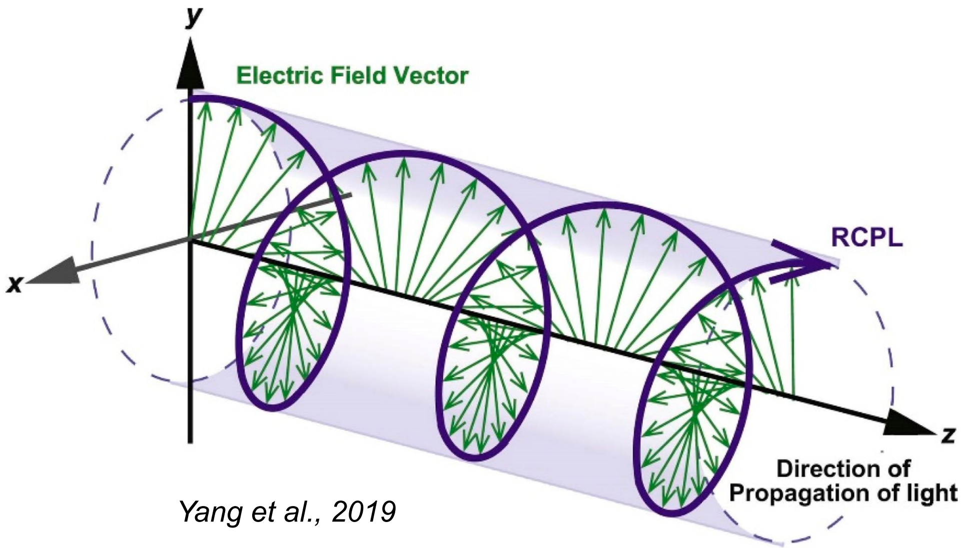
	z	w0
N.p1.i	1.0465 m	24.186 um
N.p1.i	1.0465 m	22.891 um



Siegman, 1986

Methods

- To interact properly with Rb, the light needs to be circularly polarized.
- However, planar cavities can only support one linear polarization at a time.
- Therefore, we will build a non-planar cavity that allows for circularly polarized light.
- Basic idea: non-planar cavities mix the linear polarizations.



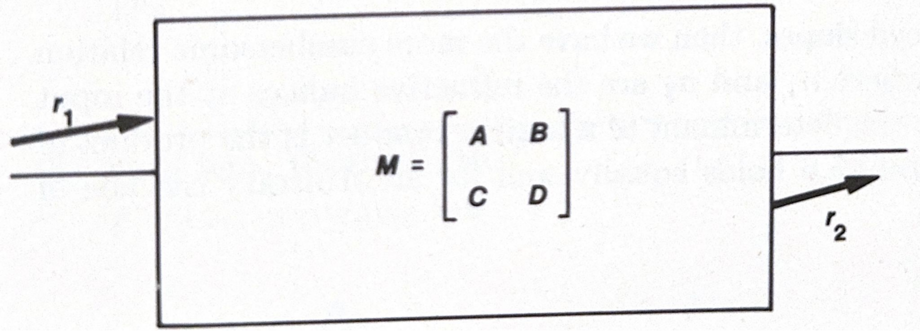
Yang et al., 2019

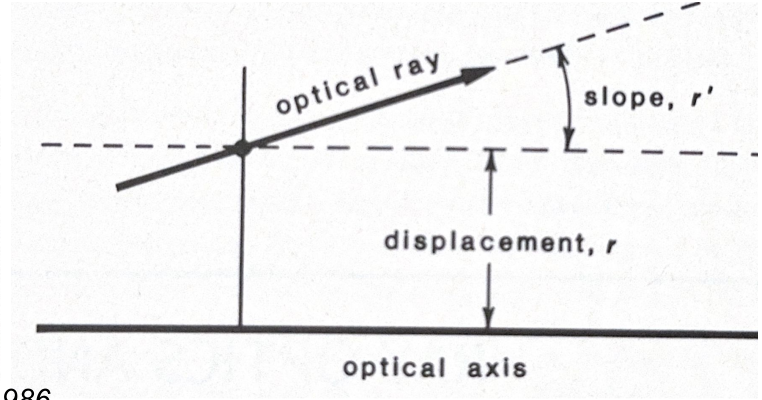
Methods

- Unfortunately, Finesse 3 does not work well with non-planar cavity designs.

- Thus, we need to consider ray transfer (ABCD) matrices.

- In the 2d case we get a 2x2 matrix.





Siegman, 1986

Methods

- In 3d, however, an additional axis introduces more elements, resulting in a 4x4 matrix.
- To do this, rotate into the frame of optical element, apply normal transformation, then reverse rotate back to original frame.
- To get the overall matrix, do this in series for all mirrors and multiply them in order.
- Off diagonal elements are hard to calculate as they involve the x-y cross-term effects.

$$\begin{bmatrix} x_2 \\ x_2' \\ \hline y_2 \\ y_2' \end{bmatrix} = \left[\begin{array}{cc|cc} A_{xx} & B_{xx} & A_{xy} & B_{xy} \\ C_{xx} & D_{xx} & C_{xy} & D_{xy} \\ \hline A_{yx} & B_{yx} & A_{yy} & B_{yy} \\ C_{yx} & D_{yx} & C_{yy} & D_{yy} \end{array}\right] \begin{bmatrix} x_1 \\ x_1' \\ \hline y_1 \\ y_1' \end{bmatrix}$$

$$\left[\begin{array}{c|c} \boldsymbol{M}_{xx} & \boldsymbol{M}_{xy} \\ \hline \boldsymbol{M}_{yx} & \boldsymbol{M}_{yy} \end{array}\right] = \begin{bmatrix} \text{overall } 4\times 4 \\ \text{matrix product} \end{bmatrix}$$

Siegman, 1986

Future Work

- Finalize mirror spacing and radius of curvature
- Advance on non-planar cavity design
- Coordinate with suppliers for curved mirror production
- Analyze system robustness against errors (placement, misalignment, etc.)

Acknowledgments

This work was supported by the following programs and fellowships:

- National Science Foundation Research Experience for Undergraduates (NSF REU) program
- LIGO Laboratory Summer Undergraduate Research Fellowship program (NSF LIGO)
- California Institute of Technology Student-Faculty Programs
- Victor Neher SURF Fellowship

I would like to thank my co-mentors for their invaluable help and guidance throughout this project: Daniel Grass and Professor Lee McCuller.

References

- Sander M. Vermeulen, Torrey Cullen, Daniel Grass, Ian A. O. MacMillan, Alexander J. Ramirez, Jeffrey Wack, Vincent S. H. Lee, Kathryn M. Zurek, Chris Stoughton, and Lee McCuller. Photon Counting Interferometry to Detect Geontropic Space-Time Fluctuations with GQuEST. April 2024.
- Daniel A. Steck. Classical and Modern Optics. 2024. Originally published in 2006. Available online at http://steck.us/teaching (revision 1.8.5, 25 May 2024).
- Anthony E. Siegman. Lasers. University Science Books, 1986.
- Yang, G.; Zhang, S.; Hu, J.; Fujiki, M.; Zou, G. The Chirality Induction and Modulation of Polymers by Circularly Polarized Light. *Symmetry* **2019**, *11*, 474.